

Economics 130 – 1st Sem. 2011-12
 Elements of Mathematical Economics
 Exercise 6 - ANSWERS

1. Production function: $Q = 1000 + 80L - 0.5L^2$, $P = 40, P_L = 80$

Profit $\pi =$ Total Revenue - Total Cost

$$\pi = P \cdot Q - P_L \cdot L = 40(1000 + 80L - 0.5L^2) - 80L$$

$$\frac{d\pi}{dL} = 40(80 - L) - 80 = 0$$

$$3200 - 40L = 80$$

$$40L = 3200 - 80 = 3120$$

$$\boxed{L^* = 78}$$

$$\frac{d^2\pi}{dL^2} = -40 < 0 \quad \rightarrow \therefore \text{maximum}$$

2. $Q_d = 600 / P^{0.9} \quad \rightarrow Q_d = 600P^{-0.9}$

a. Using the usual elasticity formula

$$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -0.9(600)P^{-1.9} \cdot \frac{P}{Q} = \frac{-0.9(600)P^{-1.9} \cdot P}{600P^{-0.9}} = -0.9$$

b. Using logarithms:

$$\ln Q_d = \ln 600 - 0.9 \ln P$$

$$\varepsilon = \frac{d \ln Q_d}{d \ln P} = -0.9 \quad \therefore |\varepsilon| = 0.9 > 0 \quad \text{inelastic at all points}$$

3. GDP per capita growth rate

$$GDP: \quad Y = 10e^{0.06t}$$

$$r_Y = \frac{dY / dt}{Y} = \frac{0.06(10e^{0.06t})}{10e^{0.06t}} = \boxed{0.06} \quad \text{or 6\% p.a.}$$

$$Population: \quad N = 90e^{0.025t}$$

$$r_N = \frac{dN / dt}{N} = \frac{0.025(90e^{0.025t})}{90e^{0.025t}} = \boxed{0.025} \quad \text{or 2.5\% p.a.}$$

$$GDP \text{ per capita} = \frac{GDP}{Population}$$

$$r_{y/n} = r_Y - r_N = 0.06 - 0.025 = \boxed{0.035} \quad \text{or 3.5\% p.a.}$$

4. Rate of growth of production

$$Y = 25(2)^{-5t}, \quad A = 100e^{0.04t} \quad \text{note: } \frac{d}{dt}b^t = b^t \ln b$$

$$r_Y = \frac{dY/dt}{Y} = \frac{(0.5)25(2)^{-5t} \ln 2}{25(2)^{-5t}} = 0.5 \ln 2 = \boxed{0.3465}$$

$$r_A = \frac{dA/dt}{A} = \frac{0.04(100e^{0.04t})}{100e^{0.04t}} = \boxed{0.04}$$

Production = Yield x Area $\rightarrow Q=Y*A$

$$r_Q = r_Y + r_A = 0.3465 + 0.04 = \boxed{0.3865}$$

5. $A = Ve^{-rt} = V = 1000e^{1.4\sqrt{t}}(e^{-0.10t})$ i.e. V continuously discounted at 10% per annum

$$\ln A = \ln 1000 + 1.4t^{0.5} - 0.10t$$

$$\frac{d}{dt}(\ln A) = \frac{1}{A} \frac{dA}{dt} = 0.7t^{-0.5} - 0.10$$

$$\frac{dA}{dt} = A(0.7t^{-0.5} - 0.10) = 0 \quad \text{equate to 0 for a maximum}$$

$$\text{Since } A \neq 0, \quad 0.70t^{-0.5} - 0.10 = 0 \quad \Rightarrow \quad t^{-0.5} = 0.10/0.7 = 0.1428$$

$$t^{0.5} = \frac{1}{0.1428} = 7 \quad \Rightarrow \quad t = 7^2 = \boxed{49 \text{ years}}$$